# MAGNETIC CIRCUIT DESIGN

## 1. MAGNETIC CIRCUIT ANALYSIS

### 1-1. Basic calculation method

The basic calculation method of a magnetic circuit is the same as is used in a basic electrical analysis using Ohm's Law.
The total magnetic flux ø (analogous to electric current), magnetomotive force F (analogous to voltage), and magnetic reluctance R (analogous to electrical resistance) are related as shown in below.

$$\text{Total magnetic flux } ø = \frac{\text{Magnetomotive force F}}{\text{Magnetic reluctance R}} \quad \text{............(1)}$$

In magnetic circuit calculations, it is more common to use the magnetic permeance P, which is the reciprocal to reluctance R.
Using permeance instead of reluctance, the total flux equation is changed as shown in below.

Total magnetic flux ø

$$= \text{Magnetomotive force F} \bullet \text{Permeance P} \quad \text{............(2)}$$

The permeance P is a function of the magnetic circuit length L, magnetic circuit cross sectional area A, and magnetic permeability μ.

$$\text{Permeance P} = \frac{\text{Permeability } \mu \bullet \text{Cross sectional area A}}{\text{Magnetic circuit length L}} \quad \text{..........(3)}$$

This means that when the length is shorter and the magnet area and permeability are greater, the permeance is greater.
Conversely the reluctance is reduced.

The total magnetic circuit permeance Pt is considered as a total of any air gap permeance Pg defined as a reciprocal of magnetic reluctance in the air gap, and the sum of leakage permeance Pf, defined as the reciprocal of the magnetic reluctance of the leakage flux paths at the poles.

$$Pt = Pg+Pf = Pg+Pf_1+Pf_2+ \text{................} +Pf_n \quad \text{............(4)}$$

### 1-2. Magnetomotive force loss coefficient f

The magnetomotive force loss coefficient f is the ratio of the total magnetomotive force Ft and the magnetomotive force in the air gap Fg for a given magnetic circuit.

$$\text{Magnetomotive force loss coefficient f} = \frac{Ft}{Fg} \quad \text{............(5)}$$

The total magnetomotive force Ft in the magnetic circuit is determined as the product of the magnetic field strength Hd at the operating point, and the length of the magnet Lm.

The magnetomotive force in the air gap Fg is given as the product of the magnetic field strength of the air gap Hg(=Bg), and the length of the air gap Lg.
Thus equation (5) becomes as follows.

Magnetomotive force loss coefficient f

$$= \frac{Hd \bullet Lm}{Hg \bullet Lg} = \frac{Hd \bullet Lm}{Bg \bullet Lg} \quad \text{............(6)}$$

### 1-3. Leakage coefficient σ

The leakage coefficient is the ratio of the total magnetic flux øt generated from the magnet in a given circuit and the flux found in the air gap øg.

$$\text{Leakage coefficient } \sigma = \frac{øt}{øg} \quad \text{............(7)}$$

The total magnetic flux generated in a magnet øt is given as the accumulation of flux density at the operating point Bd over the cross sectional area of the magnet Am.
And the magnetic flux in the air gap øg is given as the accumulation of flux density Bg over the area of the air gap Ag.
Equation (7) now becomes:

$$\text{Leakage coefficient } \sigma = \frac{Bd \bullet Am}{Bg \bullet Ag} \quad \text{............(8)}$$

From equation (2), we can rewrite equation (7) as follows:

$$\text{Leakage coefficient } \sigma = \frac{Ft \bullet Pt}{Fg \bullet Pg} \quad \text{............(9)}$$

Generally the magnetomotive force loss coefficient f is approximately 1, so equation (9) becomes:

$$\text{Leakage coefficient } \sigma \fallingdotseq \frac{Pg+Pf}{Pg} = 1 + \frac{Pf}{Pg}$$

Splitting Pf in each part of the magnetic leakage path, the leakage coefficient σ can be written as equation (10).

$$\text{Leakage coefficient } \sigma \fallingdotseq 1 + \frac{Pf_1+Pf_2+ \text{.......} Pf_n}{Pg} \quad \text{............(10)}$$

### 1-4. Permeance coefficient Pc

The permeance coefficient is used to design a permanent magnet application with a B-H curve. This is defined as the ratio of flux density Bd and magnetic field strength Hd of the operating point, and equation (11) becomes:

$$\text{Permeance coefficient Pc} = \frac{Bd}{Hd} \quad \text{............(11)}$$

The relationship is shown in figure 1 below:

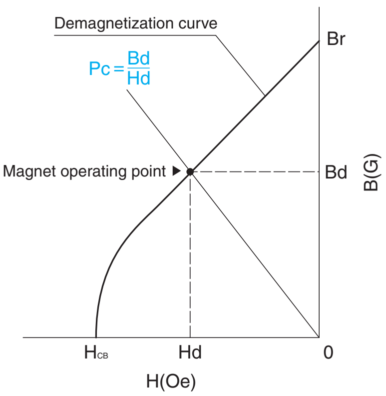


Fig. 1

• In order to make calculation formulas simple, the CGS units are applied in this section. If you need to convert the value in SI units, please refer to "Conversion Table of SI and CGS units".

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### 1-4-a. Permeance coefficient in a magnetic circuit

The permeance coefficient in a magnetic circuit can be rewritten from equations (6) and (8), and then they are placed back into equation (11).

Equation (6) therefore,

$$Hd = \frac{Bg \bullet Lg}{Lm} f \quad ............(12)$$

Equation (8) therefore,

$$Bd = \frac{Bg \bullet Ag}{Am} \sigma \quad ............(13)$$

Then,

$$Pc = \frac{Lm \bullet Ag}{Am \bullet Lg} \bullet \frac{\sigma}{f} \quad ............(14)$$

### 1-4-b. Permeance coefficient of a magnet in an open circuit

When a magnet is used in an open circuit, the permeance coefficient is greatly affected by the magnet shape. Therefore it is extremely difficult to calculate the exact value.
The following figures of the cylindrical type magnet (the relationship graph of the dimension ratio and permeance coefficient) can be used to approximate the permeance.

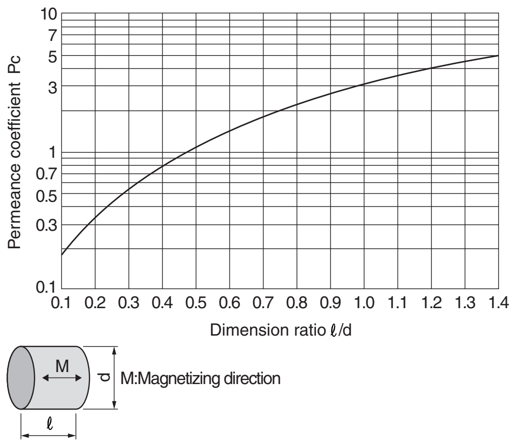



Fig. 2

The approximate equation is as follows:

$$Pc = 1.35\left(\frac{\ell}{d}\right)\left[\sqrt{1+\left(\frac{\ell}{d}\right)^2}+\left(\frac{\ell}{d}\right)\right] \quad ............(15)$$

### 1-5. Calculation of necessary magnet length Lm, cross sectional area Am and volume Vm

Equation (6) therefore,

$$Lm = \frac{Bg \bullet Lg}{Hd} f \quad ............(16)$$

Equation (8) therefore,

$$Am = \frac{Bg \bullet Ag}{Bd} \sigma \quad ............(17)$$

If Hd, Bd, Ag, Lg, f and $\sigma$ are provided, Lm and Am can be found using equations (16) and (17). The required magnet volume Vm is calculated as follows:

$$Vm = Am \bullet Lm = \frac{Bg^2 \bullet Ag \bullet Lg \bullet \sigma \bullet f}{Bd \bullet Hd} \quad ............(18)$$

The necessary magnet volume is inversely proportional to the energy product at a particular operating point.

### 1-6. Method to determine the leakage coefficient $\sigma$, and magnetomotive force loss coefficient f

### 1-6-a. Experimental method

With a search coil

1. A search coil was winding around the magnet to measure the gap flux.
2. Bd is calculated from total flux øg/magnet sectional area Am.
3. Determine Hd using the B-H demagnetization curve.
4. Measure Bg, Ag, Lg, Am and Lm.
5. Calculate f and $\sigma$ using equations (6) and (8).

Without a search coil

1. Hypothesize f (generally 1.0 to 1.2).
2. Measure Lm, Bg and Lg.
3. Calculate Hd is using equation (12).
4. Determine Bd from the B-H demagnetization curve.
5. Measure Am and Ag, and calculate $\sigma$ using equation(8).

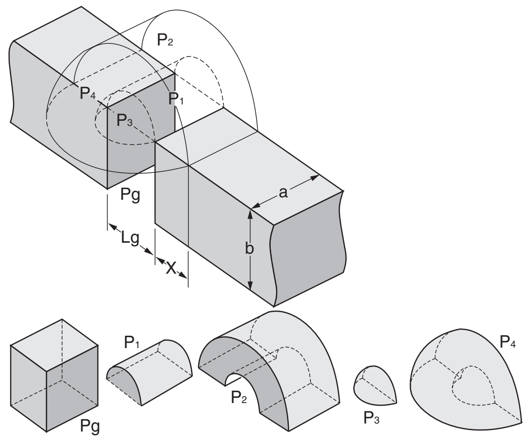
### 1-6-b. Calculating leakage $\sigma$

The leakage coefficient is calculated as follows:

Determine Pg and $Pf_1$ to $Pf_n$ by calculation (refer to the example shown in below) and calculate $\sigma$ using equation (10).

$$\text{Equation (10): } \sigma \fallingdotseq 1+\frac{Pf_1+Pf_2+......+Pf_n}{Pg}$$

(Example)
An example of the calculated permeance for a magnetic circuit is shown in below.




1. Permeance of the space gap

$$Pg = \mu_0 \frac{a \bullet b}{Lg}$$

2. Permeance of half cylinder portion

$$P_1 = 0.264\mu_0 \bullet a$$

3. Permeance of the hollow half cylinder portion

$$P_2 \simeq \frac{0.64\mu_0 \bullet a}{(Lg/x+1)} \text{ or } P_2 = \frac{\mu_0 \bullet a}{\pi} \ell n\left(1+\frac{2X}{Lg}\right) \quad [Lg<3x]$$

4. Permeance of the quadsphere portion

$$P_3 = 0.077\mu_0 \bullet Lg$$

5. Permeance of the hollow quadsphere portion

$$P_4 = \frac{\mu_0 \bullet x}{4}$$

By combining the permeance of these respective portions, the approximate leakage coefficient in the circuit can be found with using equation (10).

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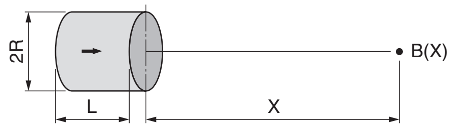
## 2. EQUATION BY DETERMINING THE MAGNETIC FLUX DENSITY AT A DISTANCE X FROM THE MAGNET AT THE CENTER LINE, B(X)

When the operating point Bd is above the knee point of the B-H curve, the magnetic field distribution on the outside of the magnet, assuming the space of length X and the same cross section shape with permeability equal to the magnet, is similar to the magnetic field generated by the closed circuit current on the outer surface.

The equations for three typical magnet shapes and some applications are shown in below. (These formulas can be used for ferrite or rare earth magnets.)

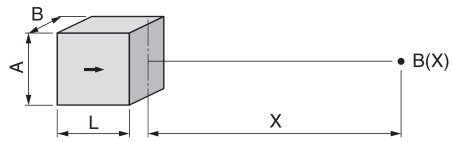
### 2-1. Cylindrical shaped




$$B(X)=\frac{Br}{2}\left[\frac{L+X}{\sqrt{R^2+(L+X)^2}}-\frac{X}{\sqrt{R^2+X^2}}\right]$$

Br: Residual flux density of the magnet
X: Distance from the surface of the magnet

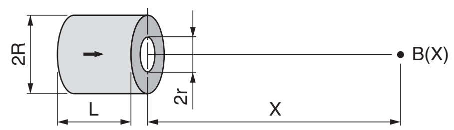
### 2-2. Block shaped




$$B(X)=\frac{Br}{\pi}\left[\tan^{-1}\frac{AB}{2X\sqrt{4X^2+A^2+B^2}}-\tan^{-1}\frac{AB}{2(L+X)\sqrt{4(L+X)^2+A^2+B^2}}\right]$$

Br: Residual flux density of the magnet
X: Distance from the surface of the magnet
The angle is radian.

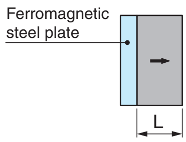
### 2-3. Ring shaped




$$B(X)=\frac{Br}{2}\left[\left(\frac{L+X}{\sqrt{R^2+(L+X)^2}}-\frac{L+X}{\sqrt{r^2+(L+X)^2}}\right)-\left(\frac{X}{\sqrt{R^2+X}}-\frac{X}{\sqrt{r^2+X^2}}\right)\right]$$

Br: Residual flux density of the magnet
X: Distance from the surface of the magnet

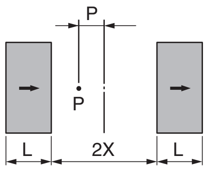
### 2-4. Ferromagnetic steel plate on the back of the magnet




The steel plate must have sufficient thickness that will be not to become saturated.

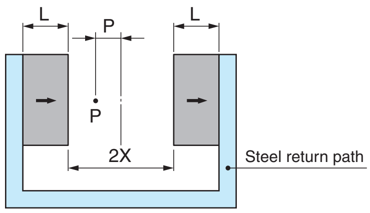
Substitute 2L for L in equations of 2-1., 2-2. and 2-3. above.

### 2-5. Calculating B(X) of same shaped two magnets when spaced at a distance of 2X




If two magnets are used as shown in above, B(X) at the center of the air gap is doubled based on equations 2-1., 2-2. and 2-3.. The B(X) at point P in the air gap is found using the same equations as above, except the sum of B(X–P) and B(X+P), where X+P and X–P are substituted into the equations for X.

### 2-6. Same shaped two magnets with a steel return path




Substitute 2L for L in the equations derived in the above 2-5..

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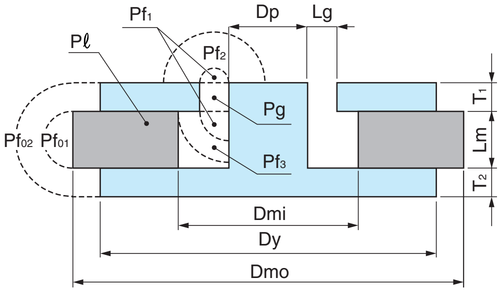
## 3. CALCULATING THE AIR GAP FLUX DENSITY FOR A LOUD-SPEAKER MAGNETIC CIRCUIT

### 3-1. Calculation method for outer magnet type

Assume the permeance can be found based on the leakage flux path shown in the figure below.
The gap permeance Pg and the leakage permeance Pf in each part can be found as follows.




Dp: Center pole diameter
$T_1$ : Top plate thickness
Dmo: Outside radius of magnet
Lm: Magnet thickness
Ag: Gap cross sectional area
Dy: Plate diameter
$T_2$ : Bottom plate thickness
Dmi: Inside radius of magnet
Lg: Gap length

$Pg = \mu_0 Ag/Lg = \pi\mu_0(Dp+Lg)T_1/Lg$
$Pf_1 = 0.264\pi\mu_0(Dp+Lg)$
$Pf_2 = \mu_0(Dp+Lg)\ell n(1+Dp/Lg)$
$Pf_3 = 2\mu_0(Dp+Lg)\ell n\{1+(Dmi-Dp-2Lg)/Lg\}$
$Pf_{01} \fallingdotseq \mu_0 0.264\pi \bullet Dmo+0.308Lm$
$Pf_{02} \fallingdotseq \mu_0 Dmo \bullet \ell n\{1+(T_1+T_2)/Lm\}+(T_1+T_2)$

$PL = \pi\mu 0\sqrt{(Dmo+Dmi)Lm/2}$

We can calculate the total permeance Pt of the whole circuit from the equations shown in above.

$Pt = Pg+3Pf_1+Pf_2+Pf_3+Pf_{01}+Pf_{02}+P\ell$

From the previous equation, we can derive an equation for the leakage coefficient.

$\sigma = Pt/Pg$

Then,

$$Pc = \frac{Lm}{Am} \bullet Pg \bullet \frac{\sigma}{f} = \frac{Lm}{Am} \bullet \frac{Pt}{f}$$

Am: Cross sectional area of magnet
f: Magnetomotive loss coefficient ≒1.1

The flux density at the magnet operating point, Bd:

$$Bd = \frac{Br}{1+\mu_{rec}/Pc}$$

$\mu_{rec}$: Recoil permeability ≒1.7

The air gap magnetic flux density, Bg:

$$Bg = \frac{Am \bullet Bd}{Ag} \bullet \frac{1}{\sigma}$$

The bottom plate must have sufficient thickness $T_2$ not to become saturated. This is determined as follows:

The leakage coefficient as viewed from the bottom plate is:

$\sigma p = (Pg+3Pf_1+Pf_2+Pf_3)/Pg$

The total magnetic flux passing through the bottom plate is:

$\phi p = Bg \bullet Ag \bullet \sigma p$

Assuming the saturated flux density of the bottom plate is 15kG, the thickness of the bottom plate $T_2$ should be:

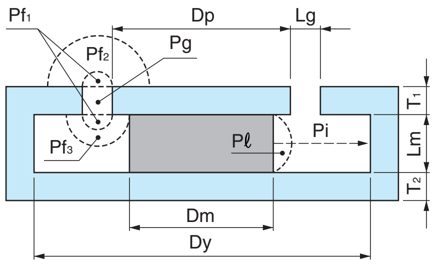
$15000\pi \bullet Dp \bullet T_2 \geqq \phi p$

Then,

$T_2 \geqq \phi p/15000\pi \bullet Dp$

### 3-2. Calculation method for inner magnet type

Assume the permeance can be found based on the flux leakage path shown in figure below in the same way as for the outer magnet type.
The gap permeance Pg and the leakage permeance Pf in each part can be found as follows:




Dp: Center pole diameter
$T_1$ : Top plate thickness
Dm: Diameter of magnet
Ag: Gap cross sectional area
Dy: Inside plate diameter
$T_2$ : Bottom plate thickness
Lm: Magnet thickness
Lg: Gap length

$Pg = \mu_0 Ag/Lg = \pi\mu_0(Dp+Lg)T_1/Lg$
$Pf_1 = 0.264\pi\mu_0(Dp+Lg)$
$Pf_2 = \mu_0(Dp+Lg)\ell n(1+Dp/Lg)$
$Pf_3 = \mu_0(Dp+Lg)\ell n\{1+(Dp-Dm)/Lg\}$
$P\ell = \pi\mu 0\sqrt{Dm \bullet Lm/2}$ $[P\ell>Pi]$
or
$Pi = \pi\mu_0 Lm(Dm+Dy)/4(Dy-Dm)$ $[Pi>P\ell]$

Then the total permeance Pt is:

$Pt = Pg+2Pf_1+Pf_2+Pf_3+P'$

P' is $P\ell$ or Pi, whichever is lager.

The calculation process of the leakage coefficient $\sigma$, permeance coefficient Pc, flux density at the operating point Bd, and air gap flux density Bg is the same as was shown for the outer magnet type.
The plate thickness $T_1$ and $T_2$ required for the plate not to become saturated is shown in below, and the calculation process is the same as for the outer magnet type.

The leakage coefficient is:

$\sigma p = (Pg+2Pf_1+Pf_2+Pf_3+P\ell \text{ or } Pi)/Pg$

The total magnetic flux $\phi p$ through the plate is:

$\phi p = Bg \bullet Ag \bullet \sigma p$

Assuming the saturated flux density of the plate is 15kG, $T_1$ and $T_2$ should be:

$T_1, T_2 \geqq \phi p/15000\pi \bullet Dm$

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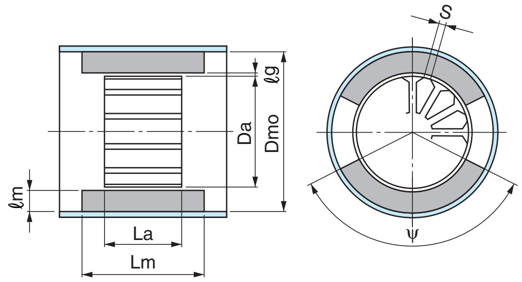
## 4. CALCULATION OF THE EFFECTIVE MAGNETIC FLUX IN A MOTOR

There are many variables that change the magnetic circuit of a motor greatly, such as the number of rotor slots, slot geometry, motor housing thickness, etc.
Therefore the following calculation shows only the basic concepts




Dmo: Outside radius of magnet
Lm: Magnet axial length
ψ: Magnet angle(radians)
ℓm: Magnet pole thickness

Da: Armature diameter
La: Armature length
S: Slot width
ℓg: Air gap length

1) Cross sectional area Am of the magnet is as follows:

$$Am=\frac{1}{2}\psi Lm(Dmo-\ell m)=\frac{1}{2P}\alpha_i \pi Lm(Dmo-\ell m)$$

$\alpha_i = p\psi/\pi$: An arc rate of a pole, P: The binary number of poles

2) Magnet equivalent value shape ratio $\ell/d$

$$\frac{\ell}{d}=\frac{\ell m}{\sqrt{4Am/\pi}}$$

3) Leakage permeance coefficient Pi and PL

• Permeance coefficient of the magnet alone $P_i$

$$P_i=\zeta\left(\frac{\ell}{d}\right)\left[\sqrt{1+\left(\frac{\ell}{d}\right)^2}+\frac{\ell}{d}\right];\zeta=\frac{1}{5}(12-p)$$

• Permeance coefficient of the magnet alone (including the yoke) $P_L$

$$P_L=\xi\left(\frac{2\ell}{d}\right)\left[\sqrt{1+\left(\frac{2\ell}{d}\right)^2}+\frac{2\ell}{d}\right];\xi=1.3\sim1.4$$

4) Carter coefficient $k_c$

$$k_c=\frac{ts}{ts-\gamma_c \ell g};\ \gamma_c=\frac{(S/\ell g)^2}{5+(S/\ell g)};\ ts=\frac{\pi Da}{Sn}$$

Where,
$t_s$: tooth pitch
Sn: Number of slots

5) Effective permeance coefficient Pu

$$P_u=\frac{\ell m}{Am}\bullet\frac{Ag}{k_c \ell g};\ Ag=\frac{1}{2}\psi k_i La(Da+\ell g);\ k_i\simeq 0.97$$

Where,
Ag: Air gap cross sectional area
$k_i$: Lamination factor in the armature

6) Leakage coefficient $\sigma$

$\sigma = 1+P_i/P_u,\ \ \sigma P_u = P_u+P_i$

7) Magnetic flux at the pole (Effective magnetic flux) $\o_g$

$$\o_g=0.95Br\frac{Am\bullet P_u}{\mu_{rec}+\sigma P_u}$$

Where,
$\mu_{rec}$: Recoil permeability

The above calculation method gives no special consideration to saturation of the magnetic circuit. In reality there can be varying degrees of saturation in the motor housing and rotor.
When comparing øg calculated above with the actually measured value, if the actually measured value is smaller than the calculated value, the motor circuit can be considered to have some saturation in one or more of the components.

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# CONVERSION TABLE OF SI AND CGS UNITS

View in conversion table:

If the conversion ratio at the left of ▶ sign is multiplied by the column of SI unit system, the value becomes in CGS system.

If the conversion ratio at the right of ◀ sign is multiplied by the column of CGS unit system, the value becomes in SI system.

| Terms | Symbol | SI units | | Conversion(Multiplication) | | CGS units | |
|---|---|---|---|---|---|---|---|
| | | Unit's name | Symbol | SI to CGS | CGS to SI | Unit's name | Symbol |
| Magnetic flux | ø | Weber | Wb | $10^{8}$▶ | ◀$10^{-8}$ | Maxwell | Mx |
| Flux density | B | Tesla | T | $10^{4}$▶ | ◀$10^{-4}$ | Gauss | G |
| Magnetic field strength | H | Ampere/meter | A/m | $4\pi \times 10^{-3}$▶ | ◀$10^{3}/4\pi$ | Oersted | Oe |
| Intensity of magnetization | M | Ampere/meter | A/m | $10^{-3}$▶ | ◀$10^{3}$ | Gauss | G |
| Magnetic polarization | J | Tesla | T | $10^{4}/4\pi$ ▶ | ◀$4\pi \times 10^{-4}$ | Gauss | G |
| Magnetomotive force | Fm | Ampere | A | $4\pi \times 10^{-1}$▶ | ◀$10/4\pi$ | Gilbert | Gi |
| Force | F | Newton | N | $10^{5}$▶ | ◀$10^{-5}$ | Dyne | dyn |
| Permeability | µ | Henry/meter | H/m | $10^{7}/4\pi$ ▶ | ◀$4\pi \times 10^{-7}$ | — | |
| Permeability, vacuum | $\mu_0$ | $4\pi \times 10^{-7}$ Henry/meter | H/m | | | 1 | |
| Reluctance | Rm | 1/Henry | $H^{-1}$ | $4\pi \times 10^{-9}$▶ | ◀$10^{9}/4\pi$ | Gilbert/Maxwell | Gi/Mx |
| Permeance | P | Henry | H | $10^{9}/4\pi\,^{9}$▶ | ◀$4\pi \times 10^{-9}$ | Maxwell/Gilbert | Mx/Gi |
| Energy product | BH | Joule/metric$^3$ | J/m$^3$ | $4\pi \times 10$▶ | ◀$10^{-1}/4\pi$ | Gauss • Oersted | G • Oe |
| | | | | $10$▶ | ◀$10^{-1}$ | erg/cm$^3$ | erg/cm$^3$ |

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