Appendix 1
S-parameter Basics
A1-1 Definition
The S-parameter (Scattering parameter) expresses device characteristics using the degree of scattering when an AC signal is considered as a wave. The word "scattering" is a general term that refers to reflection back to the source and transmission to other directions. Figure A1-1 shows an optical analogy. The word "degree" indicates the amount of attenuation or amplification, which is measured using the square root of the electric power. It is possible to understand all linear characteristics of the device based on the degree of scattering (S-parameter).

Figure A1-1 Conceptual diagram of S-parameter (optical analogy)

Transmission $b_2$

Incidence $a_1$

Reflection $b_1$

Object $S_{11}=b_1/a_1$, $S_{21}=b_2/a_1$

The input and output ports of a device are numbered and the S-parameter that is "Incident at port $j$ → Detected at port $i$" is described as $S_{ij}$. Reflection is represented as $i = j$, and transmission is described as $i \neq j$. Therefore, in an n-port device, there are S-parameters of $n^2$ pieces. When these S-parameters are aligned in a matrix form (A1-1), it is referred to as an S-matrix (Scattering matrix). For a more detailed definition, please refer to the textbooks [1] to [5].

(A1-1)

$$S = \begin{bmatrix}
S_{11} & S_{12} & \cdots & S_{1n} \\
S_{21} & S_{22} & \cdots & S_{2n} \\
\vdots & \vdots & \ddots & \vdots \\
S_{n1} & S_{n2} & \cdots & S_{nn}
\end{bmatrix}$$

The S-parameter is a ratio, so it is basically a non-dimensional parameter (no unit). However, when describing the magnitude of the S-parameter, the unit "dB" is usually used with a common logarithm. For reference, the following Table A1-1 shows some representative values.

Table A1-1 Magnitude of the S-parameter

| $|S_{ij}|$ | $20\log|S_{ij}|$ |
|----------|----------------|
| 1        | 0dB            |
| 1/\sqrt{2}| -3dB           |
| 1/10     | -20dB          |
| 1/100    | -40dB          |
| 1/1000   | -60dB          |

A1-2 Characteristics
The following characteristics are very helpful to understand S-parameter concepts.

- If a device is lossless, the S-matrix becomes unitary. Therefore, a lossless 2-port device possesses

$$|S_{11}|^2 + |S_{21}|^2 = 1 \quad \text{(Feldtkeller's formula)}$$

There is no loss, so the total amount of the scattering should be 100%. This shows the relationship, "When $S_{21}$ ($S_{11}$) is large, $S_{11}$ ($S_{21}$) is small."

- A passive 2-port device possesses

$$|S_{11}|^2 + |S_{21}|^2 \leq 1 \quad \text{(The equal sign means lossless as mentioned)}$$
Therefore, S-parameters for a passive device are not over 1 (0dB). The left side of the equation, \( PS := |S_{11}|^2 + |S_{21}|^2 \), is referred to as the “Power Scattering Ratio”. This ratio shows how much electric power the device consumes. The smaller the result for the calculation \( (0 \leq PS \leq 1) \), the greater the loss.

- The cut-off frequency indicates the boundary between the passband and stopband of a device (filter). It can also be referred to as the frequency at which \( |S_{ij}| = -3\text{dB} \) (half of the electric power passes). Therefore, if a 2-port device is lossless, \( |S_{11}| = |S_{21}| \) (this means the intersection point of the \( |S_{11}| \) graph and the \( |S_{21}| \) graph) at the cut-off frequency (refer to Figure A1-6 for an example).
- If a device is reciprocal and is not a unidirectional component such as an isolator or circulator, the S-matrix is symmetric. Therefore, \( S_{ij} = S_{ji} \).

In mixed-mode S-parameters (refer to A1-10), \( S_{cc21} = S_{cc12}, S_{cd21} = S_{dc12}, S_{dc21} = S_{cd12}, S_{dd21} = S_{dd12} \).

**A1-3 Touchstone Format**

Recently network analyzers are generally used to measure S-parameters. When the data that is to be transferred or that is used for simulation is described numerically, it is convenient to save it as a Touchstone format text file (.snp). Figure A1-2 shows an example of a Touchstone file.

![Figure A1-2 Example of Touchstone file](image)

For an explanation of the option line (# row), refer to Table A1-2 [6]. From the next row, there are numerical values that consist of multiple columns. The far left column indicates frequencies. In this example, they are DC to 6GHz (0Hz to 6000MHz). The frequency range and its interval are not ruled, but it must be arranged in increasing order. The remaining eight columns show the S-parameters at each frequency. The order for a 2-port device is \( S_{11}, S_{21}, S_{12}, \text{and } S_{22} \). Each S-parameter is described using two real numbers. (In this example, MA format is used, so the magnitude and phase are real numbers.) Therefore, there are a total of eight columns; nine columns including the frequency. In other cases than a 2-port device, the Touchstone file looks similar, but there are some differences, for example, the numerical values may be ordered in a matrix form. The description format for other than 2-port devices is almost the same with some minor differences, e.g. the order of the numerical values may be in a matrix form.

**Table A1-2 Rules of option line (# row)**

<table>
<thead>
<tr>
<th>Example</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>MHz</td>
<td>Unit of frequency</td>
</tr>
<tr>
<td>S</td>
<td>Circuit matrix type (such as S/Z/Y)</td>
</tr>
<tr>
<td>MA</td>
<td>Expressive form for complex numbers</td>
</tr>
<tr>
<td></td>
<td>MA: Magnitude / Angle (Phase)</td>
</tr>
<tr>
<td></td>
<td>RI: Real / Imaginary</td>
</tr>
<tr>
<td></td>
<td>DB: Magnitude in dB / Angle</td>
</tr>
<tr>
<td>R50</td>
<td>reference impedance/Ω</td>
</tr>
<tr>
<td>R50 refers to 50Ω</td>
<td></td>
</tr>
</tbody>
</table>

**A1-4 Reasons Why S-Parameters are Used**

As electronic devices have become faster and faster, greater emphasis has been placed on analog characteristics (SI=Signal Integrity) even in digital circuits, where in the past S-parameters were not so commonly
used. Now, S-parameters are being given more attention, and the following shows why and how S-parameters are used.

- The transfer of electric signals or power (energy) can be expressed by S-parameters, which can show such physical quantities as attenuation of a filter or transducer gain of an active device.
- When the size of a device at high-frequency is similar to the wavelength, it is necessary to consider the time difference for the location. It is easy to understand this phenomenon by using the concept of reflection and transmission. Also, the calculation can be simpler. This is why the S-matrix for a transmission line can be described by the simple formula shown in (A1-13).
- A passive device always has S-parameters (not divergence). Therefore, for example, it is valid to analyze an ideal transformer network.
- It is difficult to achieve strict termination conditions such as open and short at high frequencies. Because of resistive termination, S-parameters can be measured at high frequencies. The measured amount makes calculation easier, as there is no need to convert to Z- or Y-parameters.

### A1-5 Impedance

The characteristics of a linear 1-port (two-terminal) device can be described by one complex number, such as the impedance, admittance or reflection coefficient. The following gives an overview of these coefficients.

**Impedance** $Z$ is the ratio of the voltage between terminals and the current through terminal. The inverse of this ratio is admittance $Y$. These are complex numbers that can be described using two real numbers such as real / imaginary numbers or a polar form.

The real part of the impedance is referred to as “Resistance $R$” and the imaginary part is referred to as “Reactance $X$”. The real part of the admittance is referred to as “Conductance $G$” and the imaginary part is referred to as “Susceptance $B$”.

These are determined by two real numbers, so it is also possible to describe them in a composition circuit, e.g. capacitor and resistor (C-R) or inductor and resistor (L-R). A parallel or a series connection can be used for composition, so there are a total of four possible composition circuits (refer to Table A1-3).

$L$ and $C$ in this circuit are mere **L and C in this circuit are mere**

<table>
<thead>
<tr>
<th>Domain</th>
<th>Expression Circuit</th>
<th>$D = \tan \delta = 1/Q$</th>
<th>Interrelation</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Inductive</strong></td>
<td>$Z = R + jX =</td>
<td>Z</td>
<td>e^{j\theta}$</td>
</tr>
<tr>
<td>$X \geq 0, B \leq 0$</td>
<td>$X = \omega Ls$</td>
<td>$D = R/X = -G/B$</td>
<td>$RG = 1/(1+Q^2)$, $XB = -1/(1+D^2)$</td>
</tr>
<tr>
<td>$0 \leq \theta \leq \pi/2$</td>
<td>$X = -1/\omega Cs$</td>
<td>$D = c\omega R/G$</td>
<td>$Cs = (1+D^2)$</td>
</tr>
<tr>
<td><strong>Capacitive</strong></td>
<td>$Z = 1/(C s + jXCp)$</td>
<td>$D = -R/X = -G/B$</td>
<td>$Lp = Ls (1+D^2)$</td>
</tr>
<tr>
<td>$X \leq 0, B \geq 0$</td>
<td>$X = -1/\omega Cs$</td>
<td>$D = c\omega R/G$</td>
<td>$Cs = (1+D^2)$</td>
</tr>
<tr>
<td>$-\pi/2 \leq \theta \leq 0$</td>
<td>$X = -1/\omega Cs$</td>
<td>$D = c\omega R/G$</td>
<td>$Lp = Ls (1+D^2)$</td>
</tr>
</tbody>
</table>

In case of $D < 1$

- $RG = D^2$, $XB = -1$
- $Cs = Gp$, $Lp = Ls$
to determine the measurement mode from either \( C_p \) or \( C_s \) before measuring. However, if two independent parameters (a complex number) are acquired, it is possible to convert them after measuring by using the formulae shown in Table A1-3.

The real part of the impedance and admittance represents device loss. This is sometimes described using the ratio "\( D \)" (=\( \tan \delta \), dissipation factor) between the real and imaginary parts. The \( \delta \) is equal to the complimentary angle of the phase of either impedance or admittance. However, the value itself is rarely used. If the device is low loss, the \( \tan \delta \) is small, so it is described using a percentage. Also, "\( Q \)" (quality factor), the inverse of \( \tan \delta \) is widely used.

When an inductor and a capacitor are connected in series or parallel, it becomes a resonance circuit. The following formula shows the relationship between \( Q \) of the resonance circuit (= central frequency divided by half bandwidth, describes the resonance sharpness) and the quality factor of the inductor \( (Q_L) \) and capacitor \( (Q_C) \) when each has loss.

\[
(A1-2) \quad \frac{1}{Q} = \frac{1}{Q_L} + \frac{1}{Q_C}.
\]

Therefore, the \( Q \) of inductor / capacitor can be defined as the \( Q \) of the resonance circuit when a lossless \( (Q = \infty) \) capacitor / inductor are connected, respectively.

**A1-6 Smith Chart**

The S-parameter of a 1-port device is referred to as the reflection coefficient (in other words, S-parameters are the expansion of the reflection coefficient to more than two ports). Reflection coefficient \( \Gamma \) has the following relationship with impedance \( Z \) and admittance \( Y \), which is also a 1-port parameter.

\[
(A1-3) \quad \Gamma = \frac{Z - Z_0}{Z + Z_0} = \frac{Y_0 - Y}{Y_0 + Y}.
\]

Where the reference impedance is \( Z_0 = 1/Y_0 \).

The Smith chart has scaling of the complex plane of reflection coefficient which enables impedance to be interpreted directly. As shown in (A1-3), if including the infinite point, \( \Gamma \), \( Z \), and \( Y \) correspond one-to-one. Figure A1-3 shows this bilinear correspondence. This figure indicates that a passive device (right half of the impedance complex plane) locates inside of the Smith chart. Short \( (Z = 0 \Omega, \text{ the origin of the impedance plane}) \) is the left edge \( (\Gamma = -1) \), open \( (Z = \infty \Omega) \) is the right edge \( (\Gamma = 1) \), and \( Z = Z_0 \) is at the center \( (\Gamma = 0) \) in the Smith chart (Figure A1-4). The blue line in Figure A1-4 is "constant Q circle" for \( Q = 1 \). This is the curve that connects \( X = \pm R \) points on the Smith chart, which is the half-line whose slope is \( \pm 1 \) in the impedance plane.
As an example, a series LCR resonance circuit is plotted on Figure A1-4 in red. The frequency is not directly shown in the figure. However, the trace is on the right edge ($\Gamma = 1$, open) in low frequency. From that point, it goes clockwise when the frequency increases and reaches back to the right edge again in a high frequency. On the impedance plane, the trace is parallel to the imaginary axis through the point ($R$, 0).

The line is then converted to a circle as shown in Figure A1-3. According to Foster’s reactance theorem, reactance is an increasing function of frequency. Therefore, on the Smith chart, the movement is clockwise.

The resonance frequency of the series LCR circuit is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}.$$  

which is the point when the red line intersects with the real $\Gamma$ axis. The intersection with both constant Q circles shows the half bandwidth, $\Delta \omega = R / L$ and the Q of the resonance circuit is given by

$$Q = \frac{R_0}{\omega_0 R}.$$  

where $R_0 := \sqrt{L / C}$.

As shown in this figure, the lower the loss (the closer the red line is to the outer circumference of the Smith chart), the narrower the half bandwidth. As a result, Q becomes higher. The Smith chart is used to measure impedance (converted from the reflection coefficient) and to design matching circuits of microwave amplifiers.

**A1-7 The S-parameter for Two-Terminal Components**

The following shows how to configure 2-port circuits by arranging a two-terminal component, which is characterized by impedance. Figure A1-5 shows the two simplest types of circuit. In practical use, most circuits have a combination of these two circuits (refer to the next section for information about connections). In this sense, these two circuits are basic and the S-parameters given in SEAT are taken from these configurations.

Figure A1-5 Configuration of 2-port circuits with a two-terminal component

![Figure A1-5](image)

Theoretical formulae for the S-matrix of these configurations are given by

$$S_{\text{series-thru}} = \frac{1}{Z + 2Z_0} \begin{bmatrix} Z & 2Z_0 \\ 2Z_0 & Z \end{bmatrix},$$

$$S_{\text{shunt-thru}} = \frac{1}{Y + 2Y_0} \begin{bmatrix} -Y & 2Y_0 \\ 2Y_0 & -Y \end{bmatrix}.$$  

So if the component is lossless, the condition to be cut-off is $Z = j2Z_0$ for series-through and $Y = j2Y_0$ for shunt-through. The following shows an example using ferrite beads (Figure A1-6). The cut-off frequency in the low range is around 5MHz. At that frequency, it can be seen that $|Z| = 100 \Omega$, which is double the reference impedance. If the reference impedance is not $50 \Omega$, the result will be different (refer to Figure A1-7).
The above shows the ideal relationship between impedance and the S-parameter, but when using actual measurement data, care must be taken. This is because the interaction with the GND exists for actual measurements even when the configuration is the same as Figure A1-5.

**A1-8 Cascade Connection**

Cascade connection is most often used to construct circuits. The S-matrix $S_{MN}$ for the whole cascade connection of a 2-port device M, and a 2-port device N (the S-matrix for M and N is $S_M$ and $S_N$, respectively) is given by

\[
S_{MN} = \frac{1}{1 - S_{N11}} \begin{bmatrix} S_{M11} - S_{N11} | S_M | & S_{M12} S_{N11} \\ S_{N21} S_{M21} & S_{N22} - S_{M22} | S_N | \end{bmatrix}
\]

Where the reference impedance for the connected ports should be equal to each other, the calculation for cascade connection usually uses an F- or T-matrix. However, the formula (A1-8) is convenient when the device is described by using S-parameters and therefore does not require conversion.

The input impedance (described by the reflection coefficient $\Gamma_{IN}$) when the port 2 of a device M is terminated can be calculated in case that the device N is a 1-port device ($S_{N11}$ is its reflection coefficient). Therefore, the reflection coefficient $\Gamma_{IN}$ is the $(1,1)$ element of (A1-8),

\[
\Gamma_{IN} = \frac{S_{M11} + S_{M12} S_{N11} S_{M21}}{1 - S_{M22} S_{N11}}.
\]

**A1-9 Characteristics of Transmission Lines**

Group delay time and characteristic impedance are the characteristics of a transmission line or a filter, which is considered as a transmission line.

Group delay time $t_{GD}$ using the phase of $S_{21}$ is defined by

\[
t_{GD} = -\frac{\partial}{\partial \omega} \text{arg} S_{21}.
\]

If this value is not flat to frequency, a signal that contains multiple frequency components such as a digital waveform will be distorted. On the other hand, Characteristic impedance $Z_0$ can be calculated using the Open / Short method ($Z_\alpha = \sqrt{Z_{open} \cdot Z_{short}}$, where $Z_{open}$ and $Z_{short}$ is the input impedance terminated by open and short at the other port, respectively). The input impedance terminated by open or short can be calculated using $S_{N11}=\pm1$ in (A1-9).

As a result, using S-parameters, the characteristic impedance $Z_0$ can be expressed by

\[
\frac{Z_\alpha}{Z_0} = \sqrt{(1 + S_{11} + S_{22} + |S_1|)(1 + S_{12} - S_{11} - S_{22} + |S_1|)(1 - S_{11} - S_{22} + |S_1|)}.
\]

This calculation is the same as image impedance. This formula is based on the Open / Short method, so it is necessary to keep in mind that accurate calculations can only be made when the frequency is lower than a quarter of the wavelength $\lambda/4$. If a coupled line can be decomposed to independent lines by using the adequate
modes (for more information about "mode", please refer to A1-11), it is possible to use (A1-11) for each mode. In addition, the characteristic impedance can also be measured by TDR (Time Domain Reflectometry) using data dependant on time (location). S-matrix $S_t$ for a lossless transmission line is given by

$$
(A1-12) \quad S_t = \frac{1}{1 - \rho^2 e^{-j\theta}} \begin{bmatrix} \rho (1 - e^{-j\theta}) & (1 - \rho^2) e^{-j\theta} \\ (1 - \rho^2) e^{-j\theta} & \rho (1 - e^{-j\theta}) \end{bmatrix}.
$$

Where $\rho := (Z_{00} - Z_0)/(Z_{00} + Z_0)$, $\theta := \omega t$ and $t$ is the electrical length (Unit: Time). If the characteristic impedance of the transmission line is selected for the reference impedance, then $\rho = 0$, and (A1-12) can be simplified to

$$
(A1-13) \quad S_t = \begin{bmatrix} 0 & e^{-j\theta} \\ e^{j\theta} & 0 \end{bmatrix}.
$$

Impedance matching causes no reflection ($S_{11} = 0$). The phase of $S_{21}$ is rotated according to the electrical length. If the reference impedance in (A1-13) is changed (refer to the next section), it will be returned back to (A1-12). So the denominator in (A1-12) indicates multiple reflections due to mismatching.

A1-10 Reference Impedance

Reference impedance is an important concept to understand and use S-parameters [4], [5]. Usually we simply state that $S_{21}$ is $xx$dB. However, to be exact, it should be stated that $S_{21}$ is $xx$dB when $yy\Omega$ is used as the reference impedance. The reason why the abbreviated expression can be used is that most reference impedance is $50\Omega$. Still, it is important to keep in mind that the S-parameter is a relative (normalized) value depending on a certain reference value. In other words, reference impedance is always necessary for acquiring S-parameters (whether by actual measurement or by simulation). Z-parameter (impedance) is usually described with no reference. However, it is possible to describe it using a reference value. For example, when the reference impedance is $50\Omega$, $200\Omega$ can be described as 4, and $5\Omega$ can be described as 0.1. In this case, the calculation is simple and clear. However, the calculation for the S-parameter is a little more complicated (refer to (A1-3)).

$50\Omega$ is only a reference value, so it can be changed (renormalization = change of reference impedance). Suppose the S-parameter $S$ is already determined for $50\Omega$, it is possible to transform it to S-parameter $S'$ for other reference impedances by using

$$
(A1-14) \quad S' = (S - \rho)(I - \rho S)^{-1}.
$$

Where $I$ is the identity matrix, $\rho := (Z_{00} - Z_0)/(Z_{00} + Z_0)$, $Z_0$ and $Z_0'$ are the original and new reference impedance, respectively. It is important to keep in mind that the original S-parameters require all $S_{ij}$ parameters even when you want to know only $S'_{21}$.

Figure A1-7 $Z_0$ dependency of the S-parameter of ferrite bead

The arrows for $|S_{11}|$ and $|S_{21}|$ indicate the direction of the reference impedance increase ($10\Omega \rightarrow 50\Omega \rightarrow 100\Omega$)
Figure A1-7 shows S-parameters of ferrite bead with series-through configuration using three reference impedances of 10, 50, and 100Ω. The smaller the reference impedance, the higher the attenuation; shunt-thru capacitors, which are not shown in the figure, give the opposite, namely the bigger the reference impedance, the higher the attenuation.

A1-11 Mixed-Mode S-parameters
Since the late 1990s, high-speed differential (or balanced) transmission systems have been developed for practical use in transmitting digital signals. Although this technology has been used for many years, attention has once again been focused on these systems along with high-speed clocks.

A differential transmission system is a system that uses the differential mode (refer to the additional subheading). Therefore, the S-parameter for this system needs to be handled according to the modes, which is referred to as a mixed-mode S-parameter (or modal S-parameter) [5], [7]. Original S-parameter (referred to as a single-ended S-parameter or nodal S-parameter) indicates the response for each port. On the other hand, a mixed-mode S-parameter indicates the response for the sum of two signals (common mode) or the response for the difference of two signals (differential mode).

An explanation will be given using a 4-port device that has two input ports and two output ports (Figure A1-8). Ports 1 and 3 make up one group, and ports 2 and 4 the other.

![Figure A1-8 4-port S-parameters](image)

Mixed-mode S-parameters have the following meanings.
- \( S_{ccij} \): Common mode response.
- \( S_{ddij} \): Differential mode response.
- \( S_{cdij}, S_{dcij} \): Mode conversion between differential mode and common mode.

If the device has good symmetry, the mode conversion should be zero, which means that each mode is independent.

It is possible to calculate mixed-mode S-matrix \( S' \) by using single-ended S-matrix \( S \) [5],

\[
(A1-15) \quad S' = P^{-1}SP.
\]

Where

\[
(A1-16) \quad P = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}, \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.
\]

Here, attention must be paid to the reference impedance. The reference impedance for the common mode and differential
mode is half and twice the reference impedance for the original single-ended S-parameters, respectively. For example, if the reference impedance for original S-parameters is $50\Omega$, the reference impedance for the common mode is $25\Omega$, and for the differential mode is $100\Omega$. If the other reference impedance is needed, it can be changed by using (A1-14).

Figure A1-9 shows an example of actual measurements for a common mode filter (CMF). It can be seen that this CMF attenuates the common mode signal around 100MHz, but transmits the differential mode signal with little loss in that frequency range.

References
Common Mode and Differential Mode
Suppose there are two conductors (and a GND conductor) that are parallel to each other. When the voltage and the current for each conductor are named as \( V_1, I_1 \) and \( V_2, I_2 \), the common mode voltage \( V_c \) and Current \( I_c \) and the differential mode voltage \( V_d \) and current \( I_d \) are defined as the following (International electrotechnical vocabulary in EMC, IEC60050-161:1990, JIS C0161: 1997)

- \( V_c \): Average voltage for each conductor
  \[ V_c = \frac{(V_1 + V_2)}{2} \]
- \( I_c \): Total current for each conductor
  \[ I_c = I_1 + I_2 \]
- \( V_d \): Voltage between two conductors
  \[ V_d = V_1 - V_2 \]
- \( I_d \): Half of the current difference for each conductor
  \[ I_d = (I_1 - I_2)/2 \]

The common mode indicates the sum of signals, while the differential mode displays the difference between signals. The current in differential mode flows backwards (anti-phase) through two conductors. Therefore, GND is not directly connected (for that reason, it is referred to as normal mode). On the other hand, the current in the common mode flows in the same direction (in-phase) through two conductors. Consequently, the current flows through the GND conductor (or through another area) and then returns (hence this is referred to as an earth circuit). The common mode is also referred to as asymmetrical (or vertical, as in vertical electrical current) or longitudinal, and the differential mode is also referred to as being symmetrical (or horizontal).